Accurate Transmission Line Characterization

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Abstract—**This letter introduces a new method for the characterization of transmission lines fabricated on lossy or dispersive dielectrics. The method, which is more accurate than conventional techniques, is used to examine the resistance, inductance, capacitance, and conductance per unit length of coplanar waveguide transmission lines fabricated on lossy silicon substrates.**

I. Introduction

A NEW PROCEDURE, which we call the calibration comparison method, estimates the characteristic impedance Z_o of transmission lines fabricated on lossy or dispersive dielectrics. The new method, which has been presented in conference [1], makes use of the fact that, in a thru-reflect-line (TRL) calibration, the reference impedance is equal to the characteristic impedance of the standard line [2].

For transmission lines on nondispersive, low-loss substrates, Z_o can be determined accurately by the method of [3]. An alternative estimate, using the reflection coefficient of a small resistive load [4], is accurate only at low frequencies [1], [5].

The conventional method [6], [7] for the measurement of Z_o on dispersive or lossy substrates is based upon the measurement of the scattering parameters (S-parameters) of a single section of the transmission line. The measured S-parameters are equated to the S-parameters of an ideal transmission line that, electrically, is described solely by a characteristic impedance Z_o^e and a propagation constant γ. This approximation yields the estimate [6], [7]

$$Z_o^e = Z_r \sqrt{\frac{(1+S_{11})^2 - S_{21}^2}{(1-S_{11})^2 - S_{21}^2}}, \quad (1)$$

where the S_{ij} are the S-parameters of the line measured using a reference impedance Z_r. In certain special cases, such as with a uniform coaxial line in which a length of the dielectric has been replaced by a different material, this procedure is exact. More commonly, however, the electrical discontinuity at the connection to the transmission line cannot be described simply by the change of impedance. Therefore, the impedance Z_o^e determined by (1) is only an estimate of Z_o.

The calibration comparison method introduced here is based on an approach we suggested for determining the reference impedance of a calibration [8]. In [8], estimates of the reference impedance of an S-parameter calibration are determined from the "error boxes" relating it to a second S-parameter calibration of known reference impedance Z_r. Here, the calibration of unknown reference impedance is a TRL calibration, for which Z_r is equal to Z_o [2].

Manuscript received March 30, 1993.
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IEEE Log Number 9210807.

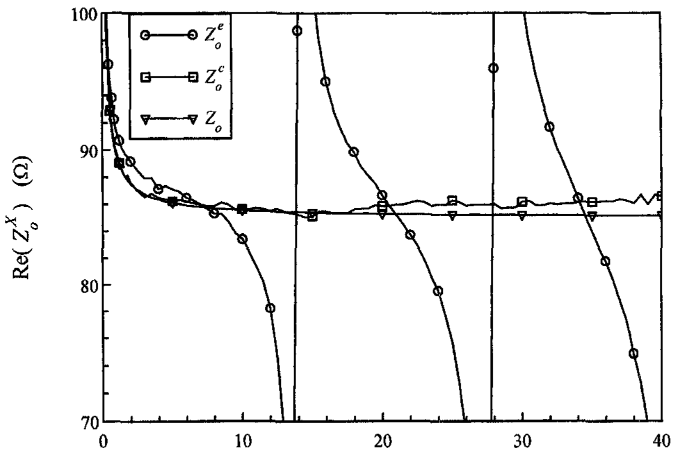

Fig. 1. The real parts of Z_o and its estimates for the CPW line fabricated on quartz.

We have presented an error analysis [1] indicating that the electrical discontinuity introduces a much smaller error in the calibration comparison method than in the conventional method, particularly when the line impedance differs greatly from the calibration reference impedance or when the line length is near a multiple of one-half wavelength. Here, we compare the two methods with lines of known Z_o to demonstrate the improved accuracy. We also apply the calibration comparison method to the electromagnetic characterization of coplanar waveguide (CPW) transmission lines fabricated on lossy silicon substrates.

II. Comparison of the Methods

We applied the two methods to the determination of the characteristic impedance of a coplanar line fabricated on quartz. The S-parameters of these lines were measured with respect to a 50-Ω multiline GaAs TRL CPW probe-tip calibration, as described in [9] and [10]. These measurements were used to perform a second multiline calibration using the quartz lines. The estimate Z_o^c was determined from this second calibration, which had a reference impedance equal to Z_o of the quartz lines. Fig. 1 compares the real part of Z_o^e from the conventional method, using the measured S-parameters of a 7.115-mm long line, to the real part of Z_o^c from the calibration comparison method and to the real part of Z_o from direct measurement [3]. The figure illustrates the large errors associated with the estimate Z_o^e from the conventional method. These contrast to the well-behaved and more accurate estimate Z_o^c from the calibration comparison method. The imaginary part of Z_o^e also deviated greatly from the directly measured values [1]. In contrast, the deviations of the imaginary part of Z_o^c were less than 0.3 Ω over most of the band and never more

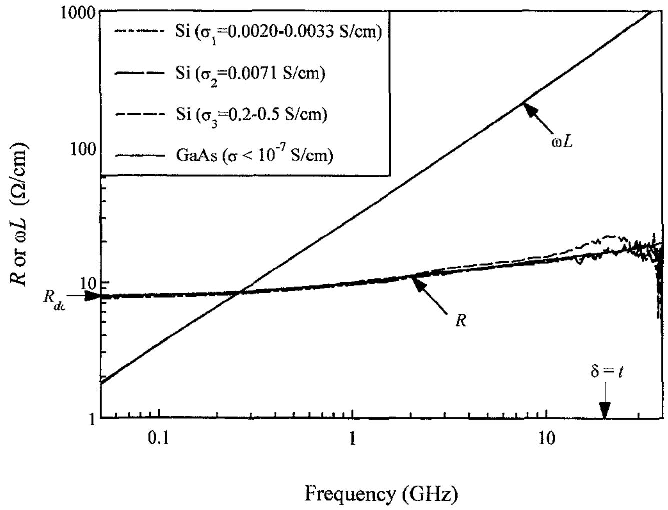

Fig. 2. R and ωL of coplanar lines fabricated on lossy silicon and semi-insulating gallium arsenide substrates. The σ_i are the approximate conductivities of the substrates as given by the manufacturer. $R_{\rm dc}$ is the measured dc resistance per unit length of the transmission lines, which were fabricated with identical conductor geometry and metal thickness. The point at which the skin depth δ is equal to the conductor thickness t is marked for reference. The four values of ωL nearly coincide.

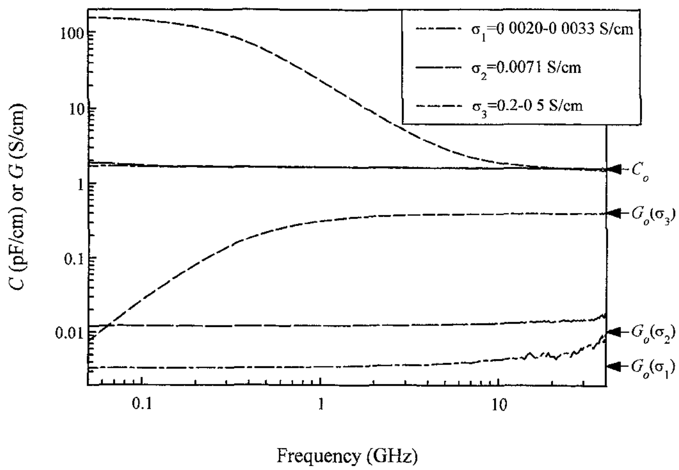

Fig. 3. C (top three curves) and G (bottom three curves) for the coplanar lines fabricated on lossy silicon substrates of Fig. 2. The σ_i are the approximate conductivities of the substrates as given by the manufacturer. C_o indicates the quasi-static capacitance per unit length of the lines calculated using a relative substrate dielectric constant of 11.7, approximately that of pure silicon. The G_o indicate the quasi-static conductance per unit length of the lines calculated from the mean value of substrate conductivity supplied by the manufacturer.

than 1.2 Ω [1]. We found that even when we used only the 7.115-mm and 0.55-mm lines in the second TRL calibration, the estimate Z_o^c did not change appreciably from the values plotted in the figure.

III. Application to Coplanar Waveguide Fabricated on Silicon

To further illustrate the utility of the calibration comparison method, we applied it to CPW lines fabricated on lossy silicon substrates. We estimated the inductance L, resistance R, capacitance C, and conductance G per unit length of the lines from Z_o^c and γ by

$$j\omega C + G \equiv \frac{\gamma}{Z_o} \approx \frac{\gamma}{Z_o^c} \tag{2}$$

and

$$j\omega L + R \equiv \gamma Z_o \approx \gamma Z_o^c. \tag{3}$$

The propagation constant γ was determined from the multiline TRL calibration [10] performed with the silicon lines. In Fig. 2, we plot R and ωL of the silicon lines alongside their values for lines of the same geometry fabricated on gallium arsenide. The figure illustrates the expected independence of R and L on the substrate. At low frequencies, R approaches the measured dc resistance $R_{\rm dc}$ indicated in the figure. L depends weakly on frequency and is nearly equal to the quasi-static value computed for lossless conductors, except at low frequencies where the internal inductance due to field penetration into the metal is significant. In Fig. 3, we plot C and G for the silicon lines. Also shown are the quasi-static capacitance C_o and conductance G_o per unit length for a coplanar line of the same dimensions, but with thin lossless conductors on a substrate with dielectric constant of 11.7, that of pure silicon. G_o was computed using the mean value of substrate conductivity supplied by the manufacturer. Fig. 3 shows that C is nearly independent of frequency and nearly equal to C_o for the lines on low-conductivity silicon but increases at the low frequencies for the silicon of high conductivity. We found this increase to depend on the dc bias on the line, an indication that it may be due to the formation of a Schottky barrier at the interface between the metal conductors and the silicon substrate. The figure also shows that G is roughly given by G_o over much of the measurement band.

IV. Conclusion

Using coplanar waveguide transmission lines of known characteristic impedance, we have illustrated the superiority of the calibration comparison method over the conventional method [6], [7]. We applied the method to coplanar lines fabricated on silicon substrates and determined R, L, C, and G from the characteristic impedance and propagation constant. R and L were seen to be nearly independent of the substrate while C and G depended strongly on the substrate. This suggests that surface impedance or other metal parameters might be accurately extracted from measurements of R and L while substrate dielectric constant or loss tangent might be accurately extracted from measurements of C and G.

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